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Minimum Independent Isolation Number of some Corona Product Graphs

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Abstract

The concept of isolation in graphs plays an important role in understanding the structural resilience and control mechanisms of networks. In this paper, we investigate the minimum independent isolation number of various corona product graphs such as $P_n \odot P_m$, $P_n \odot C_m$, $K_n \odot K_m$, $P_n \odot K_m$ and $P_n \odot S_m$. An independent isolating set I of G is defined as a subset of the vertex set V of G in which no two vertices are adjacent and when removed I from the graph along with all their neighbors, only isolated vertices are left. An independent isolating set with smallest vertices is called a minimum independent isolating set. The independent isolation number of a graph G is the order of the minimum independent isolating set of G. We compute the minimum independent isolation number of $P_n \odot P_m$, $P_n \odot C_m$, $K_n \odot K_m$, $P_n \odot K_m$ and $P_n \odot S_m$.

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1. Introduction

We consider a simple undirected connected graph G with vertex set V, and edge set E. For the graph $G = (V, E)$, we denote its order/cardinality $|V(G)|$ by the symbol n. For a vertex x inf $V(G)$, the open neighborhood set of x is denoted by $N(x)$ and is defined by $N(x) = \{y \in (V - \{x\}) : (x, y) \in E(G)\}$, while the closed neighborhood of x is denoted by the symbol $N[x]$ and defined by $N[x] = N(x) \cup \{x\}$. For a subset A of $V(G)$, the open neighborhood set of A is denoted by $N(A)$ and defined by $N(A) = \{x \in (V - A) : (x, y) \in E(G), y \in A\}$, while the closed neighborhood of A is denoted by the symbol $N[A]$ and defined by $N[A] = N(A) \cup A$. The degree of a vertex $x \in V$ is denoted by the symbol $d_G(x)$ and defined by $d_G(x) = |N(x)|$. For, a subset S of $V(G)$, $G - S$ implies the graph that is obtained from G by removing all the vertices in S and all the edges incident with a vertex in S. We also use the symbol $G[S]$ to denote the subgraph of G induced by the subset S of $V(G)$.

Let F be a family of graphs. (Caro and Hansberg, 2017) introduced the concept of an F-isolating set in a graph G. A subset I of V is called an F-isolating set if the subgraph $G - N[I]$ (obtained by removing all vertices in I and their neighbors) does not contain any graph from F as a subgraph. The smallest possible size of such a set is denoted by $\iota(G, F)$ and is called the F-isolation number of G. When $F = \{K_1\}$, then the F-isolating set becomes the same as a dominating set, and therefore $\iota(G, K_1) = \gamma(G)$, where $\gamma(G)$ represents the domination number of G. A dominating set I is called an independent dominating set if no two vertices of I are adjacent. The independent domination number of G, denoted by $i(G)$, is the smallest size among all independent dominating sets. Again, if $F = K_2$, the terms F-isolating set and F-isolation number are simply called an isolating set and isolation number, and written as $\iota(G)$. According to (Boyer and Goddard, 2024), an isolating set is equivalent to the vertex–edge dominating set, a concept earlier introduced by (Lewis et al., 2010).

In this paper, we focus on a special type of isolating set I. An independent isolating set (IIS, in short) I of G is defined as a subset of the vertex set V of G in which no two vertices are adjacent and when removed I from the graph along with all their neighbors, only isolated vertices are left. An IIS with smallest vertices is called a minimum IIS. The independent isolation number of a graph G is denoted by $\iota^i(G)$ and it is the smallest possible size of such a set. This concept was first introduced by (Lewis, 2007) under the name independent vertex–edge domination number, and has been further explored in several studies (Chen et al., 2017; Favaron and Kaemawichanurat, 2021; Zhang et al., 2023).

1.1. Survey

(Ma and Chen, 2004) proved in 2004 that $i(G) \leq \frac{n(G)}{2}$ for every connected bipartite graph G. For trees, (Favaron, 1992) showed that

$$\gamma(T) \leq i(T) \leq \frac{n(T) + |L(T)|}{3}$$

for every nontrivial tree T, where $L(T)$ denotes the set of leaves. He also described the trees for which these bounds are tight. Later, (Cabrera-Martínez et al., 2023) improved this result by proving that $\gamma(T) \leq \frac{n(T)+|S(T)|}{3}$ for any tree T with $n(T) \geq 3$, where $S(T)$ is the set of support vertices. (Cabrera-Martínez, 2024) subsequently presented an alternative and stronger version of this bound. More recently, (Krishnakumari et al., 2014) established the following bounds for the isolation number of a tree T with $n(T) \geq 3$:

$$\frac{n(T) - |L(T)| - |S(T)| + 3}{4} \leq \iota(T) \leq \frac{n(T)}{3}.$$

They also characterized the trees that exactly satisfy these bounds. In 2025, (Boyer and Goddard, 2025) studied the bounds on independent isolation in graphs. In the same year, (Hao et al., 2025) explored the independent isolation number of a tree. Several other parameters related to domination and independent domination in trees have been widely investigated in (Cabrera-Martínez, 2023; Cabrera-Martínez and Conchado Peiró, 2022; Chellali and Meddah, 2012; Dehgardi et al., 2021; Lemańska, 2004; McFall and Nowakowski, 1980; Zhang and Wu, 2022, 2024).

1.2. Applications

The minimum independent isolation set of a graph has useful applications in network security, communication systems, and resource allocation. The minimum independent isolation set of the corona product of graphs is useful in modeling hierarchical and layered networks, such as sensor–hub systems or social networks with core–periphery structures. It helps determine the smallest group of non-adjacent control or monitoring nodes needed to isolate all edges or substructures efficiently, ensuring independent supervision and optimal resource placement across both base and attached subgraphs.

1.3. Main outcome

In this paper, we compute the minimum independent isolation number for several classes of corona product graphs such as $P_n \odot P_m$, $P_n \odot C_m$, $K_n \odot K_m$, $P_n \odot K_m$ and $P_n \odot S_m$.

1.4. Organization of the paper

In the next section, we give some notations used throughout our paper. In Section 3, we present the formulae for finding the minimum independent isolation number of different types of corona graphs. In Section 4, we give the conclusion of the paper.

2. Some notations

$\iota^i(G)$	:	minimum independent isolation number of G.
IIS	:	independent isolating set.
P_n	:	path graph with n vertices.
K_n	:	complete graph with n vertices.
C_n	:	cycle graph with n vertices.
S_n	:	star graph with n vertices.

3. Minimum independent isolation number of corona product graphs

Let G_1, G_2 are two graphs with n_1 nodes, m_1 links/edges and n_2 nodes, m_2 edges, respectively. Now a corona graph $G_1 \odot G_2$ of G_1 and G_2 is made by drawing one copy of G_1 and n_1 copies of G_2 and joining the ith node point of G_1 by an edge to each node point of the corresponding copy of G_2. The number of vertices and edges of corona graphs are, respectively, $n_1 + n_1n_2$ and $m_1 + n_1m_2 + n_1n_2$.

Here, we recall two well known results of the minimum independent isolation number of Path and cycle graphs.

Theorem 1 $\iota^i(P_n) = \lceil \frac{n-1}{4} \rceil$, $n > 1$.

Theorem 2 $\iota^i(C_n) = \lceil \frac{n}{4} \rceil$.

Now, we present the computational formulae for finding the minimum independent isolation number of some corona product of graphs.

3.1. Minimum independent isolation number of $P_n \odot P_m$

Theorem 3 $\iota^i(P_n \odot P_m) = \lceil \frac{n}{2} \rceil + (n - \lceil \frac{n}{2} \rceil)\lceil \frac{m-1}{4} \rceil$, $m > 1$.

Proof. Let $P_n \odot P_m$ (for example, see the Figure 1) be a corona product graph, where $V(P_n) = \{v_1, v_2, ..., v_n\}$ and the vertices of the i^{th} of P_m are $\{v_{i,1}, v_{i,2}, ..., v_{i,m}\}, i = 1, 2, ..., n$, and $m > 1$. We also denote the subgraph of $P_n \odot P_m$ induced by the vertices $\{v_i, v_{i,1}, v_{i,2}, ..., v_{i,m}\}$, $1 \leq i \leq n$ by the symbol G_i. Now, if we remove $N[v_i]$ from G_i, then G_i becomes an edge free graph. Therefore, minimum IIS of G_i is $\{v_i\}$ and $\iota^1(G_i) = 1$. So, we can include v_1 as the first member of IIS of $P_n \odot P_m$. But, we cannot include v_2 as the second member of IIS of $P_n \odot P_m$ as $(v_1, v_2) \in E$. Also, the second copy of P_m is not vertex-edge dominated by v_1. So, we have to find the IIS of the second copy of P_m and we know that $\iota^i(P_m) = \lceil \frac{m-1}{4} \rceil$ (using Theorem 1). So, next member (after the selection of first $(1 + \lceil \frac{m-1}{4} \rceil)^{th}$ vertices of IIS) of IIS is v_3. Therefore, to find minimum IIS of $P_n \odot P_m$, we have to select maximum number of vertices of P_n as the members of IIS, and the number of these vertices (such as $v_1, v_3, v_5, ..., n$(for even) or $n-1$ (for odd)) is $\lceil \frac{n}{2} \rceil$. Again, for each attached copy of P_m corresponding to $v_i, i = 2, 4, ..., 2\lfloor n/2 \rfloor$, $\iota^i(P_m) = \lceil \frac{m-1}{4} \rceil$. In addition, $\lfloor n/2 \rfloor = n - \lceil n/2 \rceil$, if n is a natural number. Therefore, $\iota^i(P_n \odot P_m) = \lceil \frac{n}{2} \rceil + (n - \lceil \frac{n}{2} \rceil)\lceil \frac{m-1}{4} \rceil$. Hence, the result is proved. □

Note 1: $\iota^i(P_4 \odot P_3) = \lceil \frac{4}{2} \rceil + (4 - \lceil \frac{4}{2} \rceil)\lceil \frac{3-1}{4} \rceil = 2 + 2 = 4$.

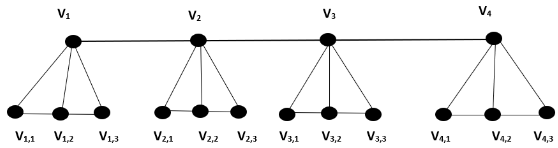

Figure 1: Corona product graph $P_4 \odot P_3$.

3.2. Minimum independent isolation number of $P_n \odot C_m$

Theorem 4 $\iota^i(P_n \odot C_m) = \lceil \frac{n}{2} \rceil + (n - \lceil \frac{n}{2} \rceil)\lceil \frac{m}{4} \rceil$.

Proof. Suppose $P_n \odot C_m$ (for example, see the Figure 2) be a corona product graph, where $V(P_n) = \{v_1, v_2, ..., v_n\}$ and the vertices of the i^{th} of C_m are $\{v_{i,1}, v_{i,2}, ..., v_{i,m}\}, i = 1, 2, ..., n$. We also denote the subgraph of $P_n \odot C_m$ induced by the vertices $\{v_i, v_{i,1}, v_{i,2}, ..., v_{i,m}\}$, $1 \leq i \leq n$ by the symbol G_i. Now, if we remove $N[v_i]$ from G_i, then G_i becomes an edge free graph. Therefore, minimum IIS of G_i is $\{v_i\}$ and $\iota^1(G_i) = 1$. So, we can include v_1 as the first member of IIS of $P_n \odot C_m$. But, we cannot include v_2 as the second member of IIS of $P_n \odot C_m$ as $(v_1, v_2) \in E$. Also, the second copy of C_m is not vertex-edge dominated by v_1. So, we have to find the IIS of the second copy of C_m and we know that $\iota^i(C_m) = \lceil \frac{m}{4} \rceil$ (using Theorem 2). So, next member (after the selection of first $(1 + \lceil \frac{m}{4} \rceil)^{th}$ vertices of IIS) of IIS is v_3. Therefore, to find a minimum IIS of $P_n \odot C_m$, we have to select maximum number of vertices of P_n as the members of IIS, and the number of these vertices (such as $v_1, v_3, v_5, ..., , n$(for even) or $n - 1$ (for odd)) is $\lceil \frac{n}{2} \rceil$. Again, for each attached copy of C_m corresponding to $v_i, i = 2, 4, ..., 2\lfloor n/2 \rfloor$, $\iota^i(C_m) = \lceil \frac{m}{4} \rceil$. In addition, $\lfloor n/2 \rfloor = n - \lceil n/2 \rceil$, if n is a natural number. Therefore, $\iota^i(P_n \odot C_m) = \lceil \frac{n}{2} \rceil + (n - \lceil \frac{n}{2} \rceil)\lceil \frac{m}{4} \rceil$. Hence, the result is proved. □

Note 2: $\iota^i(P_4 \odot C_3) = \lceil \frac{4}{2} \rceil + (4 - \lceil \frac{4}{2} \rceil)\lceil \frac{3}{4} \rceil = 2 + 2 = 4$.

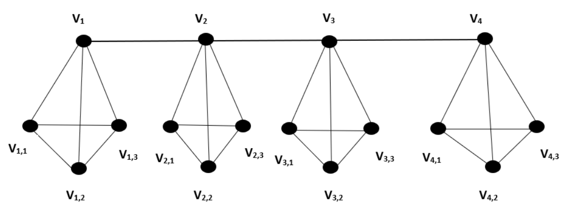

Figure 2: Corona product graph $P_4 \odot C_3$.

3.3. Minimum independent isolation number of $K_n \odot K_m$

Theorem 5 $\iota^i(K_n \odot K_m) = n$.

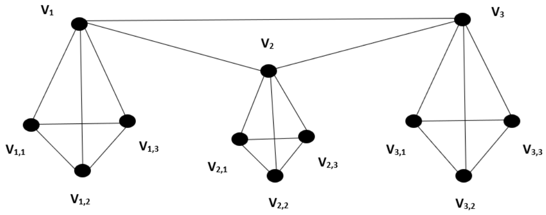

Figure 3: Corona product graph $K_3 \odot K_3$.

Proof. Suppose $K_n \odot K_m$ (for example, see the Figure 3) is a corona product graph, where $V(K_n) = \{v_1, v_2, ..., v_n\}$ and the vertices of the i^{th} of K_m are $\{v_{i,1}, v_{i,2}, ..., v_{i,m}\}, i = 1, 2, ..., n$. We also denote the subgraph of $K_n \odot K_m$ induced by the vertices $\{v_i, v_{i,1}, v_{i,2}, ..., v_{i,m}\}$, $1 \leq i \leq n$ by the symbol G_i. We know that a complete graph is vertex-edge dominated by any vertex of it. So, we can select any vertex of K_n or K_m as the 1st member of IIS of $K_n \odot K_m$. Let, v_1 is the 1st selected member of IIS of $K_n \odot K_m$. So, we cannot select any other vertices of K_n as the 2nd member of IIS as K_n is a complete graph. So, after removal $N[v_i]$ from $K_n \odot K_m$, only $n-1$ copies of K_m corresponding to the vertices $v_2, v_3, ..., v_n$. Therefore, to make edge free of these $n-1$ copies of P_m, we have to select one vertex from each $n-1$ copies of K_m as a member of IIS. Therefore, $\iota^i(K_n \odot K_m) = 1 + n - 1 = n$. Hence, the result is proved. □

Note 3: $\iota^i(K_3 \odot K_3) = 3$.

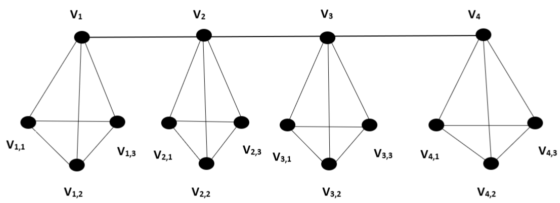

Figure 4: Corona product graph $P_4 \odot K_3$.

3.4. Minimum independent isolation number of $P_n \odot K_m$

Theorem 6 $\iota^i(P_n \odot K_m) = n$.

Proof. Suppose $P_n \odot K_m$ (for example, see the Figure 4) is a corona product graph, where $V(P_n) = \{v_1, v_2, ..., v_n\}$ and the vertices of the i^{th} of K_m are $\{v_{i,1}, v_{i,2}, ..., v_{i,m}\}, i = 1, 2, ..., n$. We also denote the subgraph of $P_n \odot K_m$ induced by the vertices $\{v_i, v_{i,1}, v_{i,2}, ..., v_{i,m}\}$, $1 \leq i \leq n$ by the symbol G_i. We know that a complete graph is vertex-edge dominated by any vertex of it. So, we can select any vertex of K_m as the 1st member of IIS of $P_n \odot K_m$. Let, $v_{1,1}$ is the 1st selected member of IIS of $P_n \odot K_m$. So, after removal $N[v_{1,1}]$ from $K_n \odot K_m$, $P_n \odot K_m$ becomes $P_{n-1} \odot K_m$. Therefore, to make edge free of $P_{n-1} \odot K_m$, we have to select one vertex from each $n-1$ copies of K_m as a member of IIS. Therefore, $\iota^i(P_n \odot K_m) = 1 + n - 1 = n$. Hence, the result is proved. □

Note 4: $\iota^i(P_4 \odot K_3) = 4$.

3.5. Minimum independent isolation number of $P_n \odot S_m$

Theorem 7 $\iota^i(P_n \odot S_m) = n$.

Proof. Suppose $P_n \odot S_m$ (for example, see the Figure 5) is a corona product graph, where $V(P_n) = \{v_1, v_2, ..., v_n\}$ and the vertices of the i^{th} of S_m are $\{v_{i,1}, v_{i,2}, ..., v_{i,m}\}, i = 1, 2, ..., n$. We also denote the subgraph of $P_n \odot K_m$ induced by the vertices $\{v_i, v_{i,1}, v_{i,2}, ..., v_{i,m}\}, 1 \leq i \leq n$ by the symbol G_i. We know that a star graph is vertex-edge dominated by its central vertex. So, we can select central vertex ($v_{1,1}$) of S_m as the 1st member of IIS of $P_n \odot S_m$. Let, $v_{1,1}$ be the 1st selected member of IIS of $P_n \odot S_m$. So, after removal $N[v_{1,1}]$ from $K_n \odot S_m$, $P_n \odot S_m$ becomes $P_{n-1} \odot S_m$. Therefore, to make edge free of $P_{n-1} \odot S_m$, we have to select one vertex from each $n-1$ copies of S_m as a member of IIS. Therefore, $\iota^i(P_n \odot S_m) = 1 + n - 1 = n$. Hence, the result is proved. □

Note 5: $\iota^i(P_3 \odot S_4) = 3$.

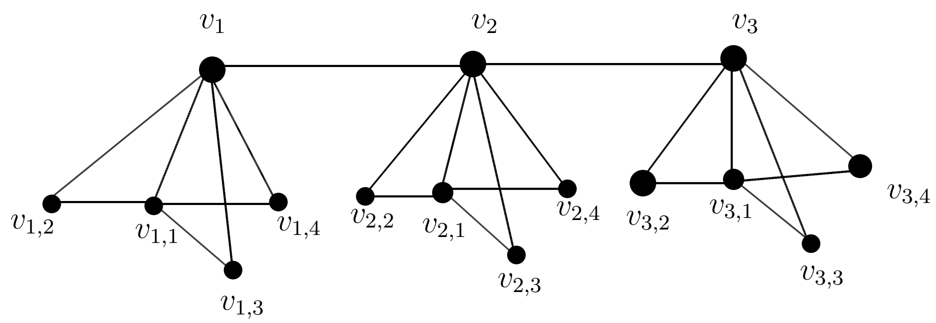

Figure 5: Corona product graph $P_3 \odot S_4$.

4. Conclusion and future scope

In this work, we have investigated the minimum independent isolation number of several important classes of corona product graphs. Although existing studies primarily focus on establishing bounds for the independent isolation number of basic graph families such as trees, bipartite graphs, and related structures, exact values for more complex constructions remain relatively unexplored. Addressing this gap, we have determined the exact minimum independent isolation number for the corona product graphs $P_n \odot P_m$, $P_n \odot C_m$, $K_n \odot K_m$, $P_n \odot K_m$ and $P_n \odot S_m$. By carefully examining the structural characteristics of each corona construction, we obtained explicit formulas and provided characterizations of their minimum independent isolating sets.

Our results highlight that the corona operation significantly influences the isolation properties of graphs. The attachment of secondary graphs to each vertex of a base graph leads to a systematic increase in the minimum independent isolation number, with the extent of this increase depending on the structural features of the attached graph.

Corona product graphs provide useful models for complex systems such as communication, biological, and social networks. The exact values obtained in this study can aid in analyzing network resilience, optimizing connectivity, and designing efficient network topologies in practical applications.

Future work may extend these results to other graph products and explore algorithmic and computational aspects of independent isolation numbers. Investigating related isolation parameters and applying the concepts to dynamic or large-scale networks also remain promising directions for further research.

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Conflict of Interest

The author declares that there is no conflict of interest regarding the publication of this manuscript.

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References

Boyer, G., Goddard, W., 2024. Disjoint isolating sets and graphs with maximum isolation number. Discrete Applied Mathematics 356, 110–116. doi:10.1016/j.dam.2024.03.012.

Boyer, G., Goddard, W., 2025. Bounds on independent isolation in graphs. Discrete Applied Mathematics 372, 143–149. doi:10.1016/j.dam.2025.01.018.

Cabrera-Martínez, A., 2023. A new lower bound for the independent domination number of a tree. RAIRO – Operations Research 57, 1951–1956. doi:10.1051/ro/2023127.

Cabrera-Martínez, A., 2024. An improved upper bound on the domination number of a tree. Discrete Applied Mathematics 343, 44–48. doi:10.1016/j.dam.2023.09.006.

Cabrera-Martínez, A., Conchado Peiró, A., 2022. On the 2-domination number of graphs. AIMS Mathematics 7, 10731–10743. doi:10.3934/math.2022607.

Cabrera-Martínez, A., Peiró, A.C., Rueda-Vázquez, J.M., 2023. Further results on the total italian domination number of trees. AIMS Mathematics 8, 10654–10664. doi:10.3934/math.2023526.

Caro, Y., Hansberg, A., 2017. Partial domination—the isolation number of a graph. Filomat 31, 3925–3944. doi:10.2298/FIL1712925C.

Chellali, M., Meddah, N., 2012. Trees with equal 2-domination and 2-independence numbers. Discussiones Mathematicae Graph Theory 32, 263–270. doi:10.7151/dmgt.1604.

Chen, X.G., Yin, K., Gao, T., 2017. A note on independent vertex-edge domination in graphs. Discrete Optimization 25, 1–5. doi:10.1016/j.disopt.2017.04.001.

Dehgardi, N., Sheikholeslami, S.M., Valinavaz, M., Aram, H., Volkmann, L., 2021. Domination number, independent domination number and 2-independence number in trees. Discussiones Mathematicae Graph Theory 41, 39–49. doi:10.7151/dmgt.2256.

Favaron, O., 1992. A bound on the independent domination number of a tree. Vishwa International Journal of Graph Theory 1, 19–27.

Favaron, O., Kaemawichanurat, P., 2021. Inequalities between the K_k-isolation number and the independent K_k-isolation number of a graph. Discrete Applied Mathematics 289, 93–97. doi:10.1016/j.dam.2020.10.010.

Hao, Q., An, X., Wu, B., 2025. The independent isolation number of a tree. Discrete Applied Mathematics 363, 1289–1299. doi:10.1016/j.dam.2024.11.021.

Krishnakumari, B., Venkatakrishnan, Y.B., Krzywkowski, M., 2014. Bounds on the vertex–edge domination number of a tree. Comptes Rendus Mathématique 352, 363–366. doi:10.1016/j.crma.2014.02.010.

Lemańska, M., 2004. Lower bound on the domination number of a tree. Discussiones Mathematicae Graph Theory 24, 165–169. doi:10.7151/dmgt.1218.

Lewis, J.R., 2007. Vertex-edge and edge-vertex domination in graphs. Ph.D. thesis. Clemson University. USA.

Lewis, J.R., Hedetniemi, S.T., Haynes, T.W., Fricke, G.H., 2010. Vertex–edge domination. Utilitas Mathematica 81, 193–213.

Ma, D.X., Chen, X.G., 2004. A note on connected bipartite graphs having independent domination number half their order. Applied Mathematics Letters 17, 959–962. doi:10.1016/j.aml.2003.09.006.

McFall, J.D., Nowakowski, R., 1980. Strong independence in graphs. Congressus Numerantium 29, 639–656.

Zhang, G., Wu, B., 2022. Domination, independent domination and k-independence in trees. Taiwanese Journal of Mathematics 26, 221–231. doi:10.11650/tjm/220201.

Zhang, G., Wu, B., 2024. 2-independent domination in trees. Journal of the Operations Research Society of China 12, 485–494. doi:10.1007/s40305-023-00457-3.

Zhang, S., Jia, T., Li, M., 2023. Partial domination of network modelling. AIMS Mathematics 8, 24225–24232. doi:10.3934/math.20231231.